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# Mystery of the Missing Antimatter

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*“Nothing is too wonderful to be true  
if it is consistent with the Laws of Nature.”*

— Michael Faraday (1791-1867)

## Abstract

The Lorentz invariant Dirac equation upon which our deepest understanding of the most fundamental Quantum Mechanics is based—exhibits perfect discrete symmetries of *Charge Conjugation* ( $C$ ), *Parity* ( $P$ ) and *Time* ( $T$ ) reversal. Not only does the Dirac equation obey these three symmetries ( $C, P, T$ ), but also all the possible combinations of these symmetries, i.e.,  $CT$ ,  $CP$ ,  $PT$  and  $CPT$ . When it comes to the  $C$ -symmetry, what this means is that—*contrary to physical and natural reality*—the Universe must contain equal portions of matter and antimatter. Obviously—this state of affairs that the Dirac equation leads to predictions that are contrary of observations—*this*—is based on the notion that the Dirac equation in its bare form as handed to us by Dirac is a correct description of reality on this front. In this chapter, we present a fundamental theoretical argument to the effect that: a symmetry violating curved spacetime version of the Dirac equation may be a perdurable solution to this long standing conundrum.

**Keywords:** Dirac equation: original, curved spacetime, antimatter, CP violation

## 1. Introduction

Perhaps persuaded and subsequently subdued by its rare, esoteric, exquisite and touchstone beauty—if one were to proceed therefrom and “religiously” believe in the verbatim predictions of the Dirac [1, 2] equation, then, they have every reason to wonder why the Universe is predominated by matter, with little—if *any*—antimatter existing naturally. The antimatter that we observe in cosmic rays is produced in the high energy collisions of matter particles. This antimatter produced in these collisions has very short life-times, lasting anything to the order of  $10^{-22}$  s. The bare Dirac equation predicts that during the moment of creation, i.e., during the so-called *Big Bang*, matter and antimatter should have been produced in equal proportions. However, today, it vividly appears that everything we see—from the smallest life forms on Earth to the largest stellar objects in the distant heavens, is made almost entirely of matter [3]. If the assumption that matter and antimatter were created in equal portions during the so-called Big Bang, surely, something must have happened to tip the balance. As such, the answer to the question of *Where did the antimatter disappear to?* is one of the greatest challenges in physics today [3]. Physicists and scientists in general, want to know what happened to the antimatter, or why we see an acute asymmetry between matter and antimatter.

While wondering of where the antimatter disappeared to, one must on the other hand smile because if indeed this antimatter were still present, you the reader would not be able to read this book because life would not have been possible as the Universe would have been nothing more than a radiation bath, because, whenever matter and antimatter meet, they annihilate to form electromagnetic radiation. So, the Universe would have been filled with photons. Hence, it is really good that antimatter did—somehow—disappear from the Universe. Be that as it may, we still have to ask why it disappeared—why? It is in asking and seeking answers to such questions that our understanding of the inner workings of the Universe deepens.

While the term antimatter was first coined by the German-born British physicist—Sir Franz Arthur Friedrich Schuster (1851–1934) [4] on 8 August 1898 in a rather whimsical letter to the journal *Nature*; in its modern form and understanding, antimatter was first predicted in 1930 by one of the finest, greatest and towering intellectual figures in English history—Paul Adrian Maurice Dirac (1902–1984) [5]. Contrary to Sir Schuster's [4] antimatter, Dirac's [5] antimatter particles have the same mass as their matter counterparts—*albeit*—with equal but opposite electric charge. For example, an Electron's antimatter counterpart is the positively charged Positron, and that of the Proton is the negatively charged anti-Proton. Matter and antimatter particles are always produced in pairs. If they come into contact, they annihilate with one another in burst of radiation leaving behind pure energy. During the first fractions of a second of the Big Bang—physicists believe that the hot and dense Universe was buzzing with particle-antiparticle pairs *popping-in-and-out* of existence in equal proportions; thus, if matter and antimatter are created and destroyed together, it seems reasonable that the Universe should today contain nothing but a leftover relic of energy. It was the American physicist—Carl David Anderson (1905–1991) [6], who first positively detected the Positron (i.e., the antimatter partner of the Electron) in the laboratory in 1932. The discovery was soon confirmed by Giuseppe Occhialini and Patrick Blackett [7], thus leading Anderson to co-receive the 1936 *Nobel Prize in Physics*.

Once it was clear that Dirac's antimatter is real, the question naturally arose—where is this antimatter? The idea that took center-stage in trying to answer this question is Sakharov's [8] hypothesis that we are going to discuss in Section 4. In a nutshell, Sakharov's [8] hypothesis imagines a Universe perfectly symmetric in its matter-antimatter constitution and due to some subtle underlying processes, this symmetry is broken. To give a vague picture of this symmetry breaking—consider for example a coin spinning on a table. Neglecting the third side of the coin which is the edge and only considering the side—head and tail, this coin when tossed can land on either on its heads or its tails. One thing that is clear is that while the coin is still spinning before it actually lands, it cannot be defined as “heads” or “tails” until it stops spinning and falls to one side. A fair and unbiased coin has a 50–50 chance of landing on either its head or tail. Thus, if a statistically large sample of these coins is spun in exactly the same way, half should land on their heads while the other half land on tails. In pretty much the same way, half of the oscillating particles in the early Universe should have decayed as matter and the other half as antimatter.

Now—in the afore-described scenario, if say a marble rolled across a table of the spinning coins, surely, it would cause an imbalance in the 50–50 ratio of heads and tails—in simple terms, the marble would disrupt the whole system insofar as the outcome is concerned. There would an imbalance of either more heads than tails or more tails than heads. In the same fashion, some unknown mechanism [e.g., Sakharov's [8] conditions] could have interfered with the initial balance of matter-antimatter that existed in the nascence of the Universe causing an imbalance that may have led to the majority of antimatter decaying to matter. This is the

predominant thinking amongst the majority of physicists as to how the present matter-antimatter asymmetry should have come about and manifested.

The thinking that the Universe should have started-off in state of balance between matter and antimatter obviously assumes that the Dirac equation in its bare form is the correct description of reality. We will argue in favor of a revision of the Dirac equation. We explore what we believe to be an equivalent of the Dirac equation on a curved spacetime. As we apply the Dirac equation in its bare form, we must be mindful that this equation does not incorporate gravity. Thus, it may very well be that, once we involve gravity, the scenario may very well be different. In actual fact, we shall do exactly that. We shall consider a set of curved spacetime Dirac equations that we proposed in the *Foundations of Physics* journal some 10 years back [9]. From these equations, we shall argue that if gravity is considered, it is possible that the Universe was created from the onset as a Universe exclusively filled with matter and with no antimatter. We think the fault squarely lies in the Dirac equation and not *Nature*—for, there is no antimatter that is missing, because it was never there to begin with.

Of the Dirac equation and its alleged fallibility, it must be said once again that this equation is one of the most successful equations in entire *History of Physics*. It is the most fundamental foundational basis of all Quantum Field Theories (QFTs) such as Quantum Electrodynamics (QED), Quantum Flavor (QFD) and Quantum Chromodynamics (QCD). It is so successful so much that, it is unimaginable for one to consider its revision—it is more like one trying to revise *Newton's Laws of Motion*. Through the centuries, *Newton's Laws of Motion* have been tested rigorously and found to be correct and the only imaginable, reasonable and logical thing one can do is to extend these into newer domains like what Einstein [10] did with the “pro-mulgation” of his *Special Theory of Relativity* (STR).

In trying to comprehend the obvious embarrassment facing the beautiful and most successful Dirac equation—*vis*—the issue of matter-antimatter asymmetry, we must remember and be fully cognizant of the fact that the Dirac equation is an equation that is based on the flat Minkowski spacetime—gravity is not present in Minkowski spacetime hence it is not present in the Dirac equation. Thus, while this equation has been successful, this obvious fragment contradiction with physical and natural reality when it comes to the missing antimatter—this, may very well be an issue whose perdurable answer is to be found in the “complete version of the Dirac equation” and this kind of equation should most certainly be the curved spacetime version of it. In-tandem with the afore-stated philosophical approach to the “Dirac equation and the missing antimatter,” the approach presented in the present chapter cannot be said to be a revision of the Dirac equation, but an extension of this equation in the domain of curved spacetime. This extended version violates  $C$ ,  $P$ ,  $T$ -symmetries and their combinations (i.e.,  $CT$ ,  $CP$ ,  $PT$  and  $CPT$ ), thus placing it on a sure pedestal to deliver an alternative view on this very important matter of the missing antimatter.

## 2. Dirac equation

In the present section, we will present an exposition of the Dirac equation and demonstrate only one of its symmetries, namely, the  $C$ -symmetry—because—this symmetry is the most relevant in our quest for the missing antimatter. As a starting point, it is important to ask “Why and how did Dirac come to discover the equation that now indelibly bares his name?” As is well known—history has recorded that Dirac embarked on the quest for the Dirac equation after it was noted that, the then and only known relativistic quantum equation—the Klein [11] and Gordon [12] Eq. (hereafter Klein-Gordon equation)—possessed both negative and positive



energy solutions and in the case of the negative energy solutions, the resulting quantum probability was negative. At any rate imaginable, negative probabilities are not only meaningless, but nonsense.<sup>1</sup> So, Dirac embarked on his esoteric quest with in mind the aim of getting reed of the negative energy solution together with them—their negative probabilities. As fate would have it, Dirac famously achieved on tackling the latter and serendipitously failed on the former—*albeit*—it is in this failure that his agile brilliance manifested and out-shined thus placing him on a very rare and unique pedestal as one of the finest minds to ever walk the face of the planet Earth.

The Klein-Gordon equation is harnessed from the usual Einstein [10] energy-momentum equation:

$$\eta_{\mu\nu} p^\mu p^\nu = \frac{E^2}{c_0^2} - p^2 = m_0^2 c_0^2, \quad (1)$$

where  $\eta_{\mu\nu}$  is usual flat Minkowski metric of spacetime,  $p_\mu$  is the four momentum,  $E$  is the energy,  $m_0$  is the rest mass of the particle and  $c_0$  is the speed of Light in *vacuo*. Upon a canonical quantization procedure is applied to Eq. (1), the resulting quantum mechanical equation—which is the Klein-Gordon equation—is:

$$\eta_{\mu\nu} \partial^\mu \partial^\nu \Psi = \nabla^2 \Psi - \frac{1}{c_0^2} \frac{\partial^2 \Psi}{\partial t^2} = \left( \frac{m_0 c_0}{\hbar} \right)^2 \Psi \quad (2)$$

where  $\Psi$  is the quantum mechanical wavefunction of the particle in question and  $\partial^\mu$  is the Laplacian operator. Because the Klein-Gordon equation quadratic in the derivatives, Dirac realized that this should be the source of the negative probabilities. So, he imagined that an equation that is linear in the derivative would to be job of not only getting reed of the ugly negative probabilities, but the negative energy solutions as-well. He was right on the former and wrong on the latter.

Direct from the beautiful world of his esoteric imagination, Dirac wrote down an equation that was *in-sync* with his desideratum, i.e., an equation that is linear in the derivatives and demanded of it that when it is “squared,” it would reduce to the well-known Klein-Gordon equation. Dirac’s *sleight of mind* worked as he managed to obtain an equation that is so rich, it is still being studied to this day. Written in its “covariant” form, the Dirac equation is given by:

$$i\hbar\gamma_\mu \partial^\mu \psi = m_0 c_0 \psi, \quad (3)$$

where:  $i = \sqrt{-1}$ ,  $\hbar$  is Planck’s normalized constant and:

$$\psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad (4)$$

is the  $4 \times 1$  Dirac four component wavefunction and the left- and right-handed bispinors  $\psi_L$  and  $\psi_R$  are such that:

<sup>1</sup> It is interesting to note that Dirac and another great mind, the American theoretical physicist—Richard Phillips Feynman (1918–1988), famously tried and monumentally failed to make sense of negative probabilities.

$$\psi_L = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} \text{ and } \psi_R = \begin{pmatrix} \psi_2 \\ \psi_3 \end{pmatrix}, \quad (5)$$

and:

$$\gamma^0 = \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (6)$$

are the  $4 \times 4$  Dirac gamma matrices where  $\mathcal{I}_2$  and 0 are the  $2 \times 2$  identity and null matrices respectively. Throughout this chapter, the Greek indices will be understood to mean:  $\mu, \nu, \dots = 0, 1, 2, 3$ , while the lower case English alphabet indices:  $i, j, k, \dots = 1, 2, 3$ .

### 3. C-symmetry of the Dirac equation

For latter instructive purposes,<sup>2</sup> we will now demonstrate that the Dirac Eq. (3) observes  $C$ -symmetry. To that end, we shall start off in the usual manner by placing the Dirac particle  $\psi$  inside an external magnetic field whose electromagnetic four vector potential is  $A_{ex}^\mu$ . So doing, Eq. (3) will transform to:

$$i\hbar\gamma_\mu(\partial^\mu + ieA_{ex}^\mu)\psi = m_0c_0\psi. \quad (7)$$

Now, we will have to switch the external magnetic field by reversing the electromagnetic four vector potential is  $A_{ex}^\mu$ , i.e.,  $A_{ex}^\mu \mapsto -A_{ex}^\mu$ . So doing, Eq. (7) will transform to:

$$i\hbar\gamma_\mu(\partial^\mu - ieA_{ex}^\mu)\psi = m_0c_0\psi. \quad (8)$$

Now, if the Dirac Eq. (3) is symmetric under electrical charge conjugation, there must exist a set of permissible mathematical operations that when applied to Eq. (8), they will lead us back to Eq. (7). The first such permissible mathematical operations is to apply the complex-operation on both-sides of Eq. (7): this complex operation will restore the sign in the coefficient of  $A_{ex}^\mu$ , i.e.,

$$-i\hbar\gamma_\mu^*(\partial^\mu + ieA_{ex}^\mu)\psi^* = m_0c_0\psi^*. \quad (9)$$

Now, in-order to revert back to Eq. (7), we need to find a set of permissible mathematical operations that will remove the complex operation on  $\gamma_\mu^*$ . This can be done because of the following *Algebra*:

$$\gamma_0\gamma_2\gamma_\mu^* = -\gamma_\mu\gamma_0\gamma_2. \quad (10)$$

The removal of the complex-operation on  $\gamma_\mu^*$  is achieved by multiplying on both-sides of Eq. (9) by  $\gamma_0\gamma_2$ , i.e.,

$$-i\hbar\gamma_0\gamma_2\gamma_\mu^*(\partial^\mu + ieA_{ex}^\mu)\psi^* = m_0c_0\gamma_0\gamma_2\psi^*, \quad (11)$$

<sup>2</sup> That is, for the benefit of the worked presented in Section 8.

and using the fact Eq. (10), it follows that Eq. (11) will reduce to:

$$i\hbar\gamma_\mu(\partial^\mu + ieA_{ex}^\mu)(\gamma_0\gamma_2\psi^*) = m_0c_0(\gamma_0\gamma_2\psi^*). \quad (12)$$

Now, Eq. (12) can be re-written as:

$$i\hbar\gamma_\mu(\partial^\mu + ieA_{ex}^\mu)\psi_c = m_0c_0\psi_c, \quad (13)$$

where:  $\psi_c = \gamma_0\gamma_2\psi^*$ , is the antiparticle. Except for the interchange of  $\psi$  with  $\psi_c$ , Eq. (13) is the same equation as the original Eq. (7), the meaning of which is that the Dirac equation is symmetric under charge conjugation, since the same law that applies to matter ( $\psi$ ) also applies to antimatter ( $\psi_c$ ).

## 4. Sakharov conditions

In 1967, exiled Soviet (dissident and) nuclear scientist—Andrei Dmitriev Sakharov (1921–1989), described three minimum conditions which are required for any baryogenesis to occur, regardless of the exact mechanism leading to the excess of baryonic matter. In his seminal paper which laid the foundations for all future attempts to explain the matter excess of the Universe, Sakharov [8] did not list the conditions explicitly. Instead, he described the evolution of a Universe which goes from a Baryon-excess ( $B$ -excess) while contracting in a *Big Crunch* to an anti- $B$ -excess after the resultant *Big Bang*. In summary, his three key assumptions are now known as they *Sakharov Conditions*, and these are [3]:

1. At least one  $B$ -number violating process.
2.  $C$  and  $B = 0$ -violating processes.
3. Interactions outside of thermal equilibrium.

These conditions must be met by any explanation in which ( $B = 0$ ) during the *Big Bang* but is very high in the present day. They are necessary but not sufficient—thus scientists seeking an explanation of the currently obtaining matter asymmetry on this basis (Sakharov conditions) must describe the specific mechanism through which baryogenesis happens. Much theoretical work in cosmology and high-energy physics revolves around finding physical processes and mechanism which fit the three Sakharov preconditions and correctly predicting the observed baryon density. Sakharov's conditions can be proven by means of Quantum Mechanics and Statistical Physics [3].

### 4.1 $B$ -number violation

We know that electric charge is a conserved quantity. The total electric charge before an interaction is always equal to the electric after the interaction. In exactly the same manner, it turns out that one can assign a charge that we call “Baryon Number” ( $B$ -number) to quarks, and experiments have demonstrated that this charge is also conserved. All quarks carry a  $B$ -number charge of  $+1/3$ , while all anti-quarks carry a  $B$ -number charge of  $-1/3$ , and everything else—i.e., Leptons, intermediate gauge Bosons, etc.; carry a  $B$ -number charge of 0. For all interaction so far observed in *Nature*,  $B$ -number is conserved and the *World* is yet to be furnished with evidence to the contrary if indeed  $B$ -number violation exists.

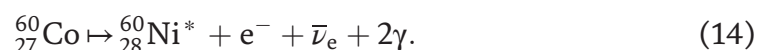
While both electric charge and  $B$ -number are conserved physical quantities, the difference, however, between these two is that,  $B$ -number conservation is considered an “Accidental Symmetry of Nature” because in constructing the Standard Model, one does not build in  $B$ -number conservation explicitly. It is not a requirement for a reasonable Standard Model, but it just so happens that when one examines the evidence from numerous experiments,  $B$ -number is always conserved somehow. Because  $B$ -number conservation is a fact of experience, it very well may be that, there exists yet to be discovered interactions where  $B$ -number conservation is not upheld. From an exploration stand-point, there simply is no reason why  $B$ -number violation should be ruled out and not be considered a possibility—especially in the light of the Sakharov conditions, hence, some physicists expect that  $B$ -number violating processes [e.g., Proton decay  $p \mapsto e^+ \pi^0$ ;  $p \mapsto \mu^+ \pi^0$ ] might exist in order to explain the matter-antimatter asymmetry.

The idea of  $B$ -number violation is central to the so-called Grand Unified Theories, i.e., a GUT in physics is a model in particle physics in which, at exceedingly high energies ( $\gtrsim 10^{16}$  GeV), the three known gauge interactions—the Electromagnetic, Weak, and the Strong interactions, or forces which define Standard Model are merged into one single unified force or interaction where in the GUT interactions are characterized by one larger gauge symmetry group. The first true GUT was proposed by Howard Georgi and Sheldon Glashow in 1974 [13] and few months later in the same journal by Jogesh Pati and Abdus Salam (1926–1996) [14]. There is currently no hard experimental evidence [15–18] that *Nature* is described by a GUT as Proton decay has not been observed. Without  $B$ -number violation, it is not possible for any system to evolve from a state with:  $B = 0$ , to a state with:  $B \neq 0$ .

## 4.2 $\mathcal{P}$ and $C\mathcal{P}$ -violation

The idea behind  $C\mathcal{P}$ -symmetry is that the equations of particle physics are invariant under mirror inversion and this leads naturally to the prediction that the mirror image of a reaction (such as a chemical reaction or radioactive decay) should occur at the same rate as the original reaction. It was not until 1956 that, along with the sacrosanct law of conservation of energy and conservation of momentum,  $\mathcal{P}$ -symmetry was believed to be one of the *Fundamental Geometric Conservation Laws of Nature*. After a careful and critical review of the existing experimental data by Tsung-Dao Lee and Chen-Ning Yang [19] revealed that while  $\mathcal{P}$ -symmetry had been verified in decays by the Strong or Electromagnetic interactions, it was untested in the Weak interaction, thus they (Lee and Yang [19]) proposed several experiments to rectify this, and simultaneously, they found a perdurable solution for this puzzle.

The first such experiment that Lee and Yang [19] proposed was the  $\beta$ -decay of Cobalt  $^{60}\text{Co}$  nuclei whose decay is as follows:



Assuming parity is violated in this interaction (Eq. (14)), the Electrons ought to be ejected differently before and after the parity transformation. The landmarking experiment was soon undertaken by Wu et al. [20] the following year in 1957.

In their experiment, Wu et al. [20] aligned the spins of a sample of  $^{60}_{27}\text{Co}$  with an external magnetic field. The sample was cooled to  $\sim 0.003$  K (Houston in Lide [21] pp. 111–115) in-order to ensure that as many nuclear spins as is possible would align. Wu et al. [20] then proceeded to count the resulting decay products of the atoms along with the direction of their propagation. After a parity transformation was applied, by means of flipping the magnetic field direction, the same measurements



were taken once again. The results were anything but surprising and ground-breaking. Rather than the Electrons being emitted in the same relative direction before and after the parity transformation, it was observed the Electrons had a “preferred” direction a certain direction and the effect was not small but pronounced—with a  $\gamma$ -ray polarization of  $\sim 60\%$ , that is to say,  $\sim 60\%$  of the  $\gamma$ -ray were emitted in one direction, whereas  $40\%$  were emitted in the other. If  $\mathcal{P}$ -symmetry was indeed conserved, Electrons would have no preferred direction of decay relative to the nuclear spin. Further, Wu et al. [20] observed that Electrons in their experiment were emitted in a direction preferentially opposite to that of the  $\gamma$ -rays. That is to say, most of the Electrons favored a very specific direction of decay, opposite to that of the nuclear spin. The clear meaning of these results is that  $\mathcal{P}$ -symmetry was violated as suggested by Lee and Yang [19], thus conclusively demonstrating that Weak interactions do indeed violate  $\mathcal{P}$ -symmetry.

Apart from the  $\text{Co}^{60}$  experiment, Lee and Yang [19] suggested as well that this same test of  $\mathcal{P}$ -violation could be made in the decay of the  $\pi^+$  and  $\mu^+$ , i.e., the reactions:

$$\begin{aligned}\pi^+ &\mapsto \mu^+ + \nu_\mu \\ \mu^+ &\mapsto e^+ + \nu_e + \bar{\nu}_\mu.\end{aligned}\tag{15}$$

As pointed out by Garwin et al. [22], parity conservation/violation can be tested in this decay chain because if  $\mathcal{P}$ -symmetry is indeed broken, there should be an asymmetry in the polarization of the Muons along the direction of motion as this can be determined from the distribution of Electrons from the decay of the Muons. Experimentalists were initially skeptical that any sizeable effect of this would be measured somehow. However, after hearing of the magnitude of the asymmetry discovered by Wu et al. [20], and liaising with her in private (see e.g., Ref. [23]), Garwin et al. [22] undertook the experiment in February of 1957, the results of which they published directly after Wu et al. [20] in the same journal with the papers stuck to each other back-to-back. Ambler et al. [24], also conducted a similar experiment which corroborated Garwin et al. [22] and Wu et al.’s [20] results. In both Garwin et al. and Ambler et al. experiments, they observed angular distribution of the Electrons was as predicted by Lee and Yang [19], thus confirming that—indeed, parity is not conserved in the Weak interactions.

Having realized that  $\mathcal{P}$ -symmetry was indeed violated—in an act of desperation—it was proposed in 1957 by the great Soviet theoretical physicist—Lev Davidovich Landau (1908–1968), that  $\mathcal{CP}$ -symmetry was the true symmetry of *Nature* to be found between matter and antimatter, i.e., this symmetry would be conserved (see e.g., Lee et al. [25]). This proposal made use of the symmetry of a quantum mechanical system emanating from the subtle structure of Hilbert Space, that, if some symmetry say  $\mathcal{S}$  can be found such that the combined  $\mathcal{PS}$ -symmetry remains unbroken, then this is the true symmetry of *Nature*. Based on this,  $\mathcal{CP}$ -symmetry was proposed as the desired symmetry to restore order and as we now know, this did not happen as Christenson et al. [26] demonstrated that  $\mathcal{CP}$ -symmetry was indeed violated. In-order to explain the matter-antimatter asymmetry, what become the central focus is the extent to which this  $\mathcal{CP}$ -symmetry occurred as it is believed (e.g., by Aaij et al. [27], amongst a host of many others) that this is not sufficient to account for matter-antimatter asymmetry.

### 4.3 Interactions outside of thermal equilibrium

The expansion [28] and ultimate accelerated expansion [29, 30] of the Universe provides the necessary platform for nonequilibrium conditions needed by the third

of Sakholov's conditions requiring that there be interactions occurring outside of thermal equilibrium [3]. Actually, during most of the history of the Universe, cosmic expansion is the only source of nonequilibrium [3]. Thermal equilibrium is a time translation invariant state in which the expectation values of all observables are constant, therefore it requires a deviation from equilibrium to evolve from:  $B = 0$ , to the desired state with:  $B \neq 0$  [3].

## 5. Current experimental and observational efforts

Gamma-ray line radiation at 511 keV is the pristine signature of Electron-Positron annihilation ( $e^+e^- \mapsto \gamma\gamma$ ) and for the past 40 years, such radiation has been known [31] to come from the Galactic Center. Weidenspointner et al. [32] of the European Space Agency (ESA) using the INTEGRAL satellite have reported a distinct radiation asymmetry in the 511 keV line emission coming from the inner Galactic disk ( $\sim 10 - 50^\circ$  from the Galactic Centre). This asymmetry Weidenspointner et al. [32] say resembles an asymmetry in the distribution of low mass X-ray binaries with strong emission at Photon energies  $>20$  keV, indicating that they may be the dominant origin of the Positrons. This observation by Weidenspointner et al. [32] may explain the origin of a giant antimatter cloud surrounding the Galactic Center. Stellar nucleosynthesis [33–35], accreting compact objects [36–39], and even the annihilation of exotic dark-matter particles [40] have long been suggested as possible causes of this 511 keV line emission. In our view, this is interesting but does not help much in the resolution of the matter-antimatter as the question will always arise as why this antimatter is not uniformly spread.

On a recent interesting note, Neri [41] presented the first and long sought evidence for  $CP$ -violation in the baryon sector as this is much closer to home where it can be linked to baryon number and ultimately to  $B$ -number violation. Neri [41] noted differences in the behavior of matter and antimatter in  $K$  and  $B$  meson decays, but not yet in any baryon decay. Such differences Neri [41] says are associated with the noninvariance of fundamental interactions under the combined  $CP$ -transformations, specifically  $CP$ -violation. In their ground breaking work, Neri [41] examined the decay products of matter and antimatter baryons (a particles containing three quarks) and looked at the spatial distribution of the resulting daughter particles within the detector. Specifically, Neri [41] looked for a very rare decay of the  $\lambda_b^0$  particle (which contains an up quark, down quark and bottom quark) into a Proton and three Pions, which contain an up quark and anti-down quark, i.e.,  $p \mapsto \pi^- \pi^+ \pi^-$  and  $p \mapsto \pi^- K^+ K^-$  final states. Based on data from  $\sim 6000$  decays with a statistical significance corresponding to 3.3-sigma level including systematic uncertainties, Neri [41] find a difference in the spatial orientation of the daughter particles of the matter and antimatter  $\lambda_b^0$ . At a 3.3-sigma level of confidence, chances of this being a just a statistical fluctuation (and not a new property of nature) is one out of a thousand. The traditional threshold for discovery is 5-sigma level of confidence, which equates to odds of one out of more than a million.

## 6. Current theoretical efforts

We are of the view that theoretical effort on the problem of matter-antimatter asymmetry can be classified into two groups, the first of which are those efforts that seek to modify or extend the Dirac equation so that it is applicable on a curved spacetime and particle physics theories that try to solve this problem from within the Standard Model. We must say that—except for the efforts presented in Refs. [9, 42]

in the case of those efforts that seek to modify or extend the Dirac equation into the domain of curved spacetimes, these efforts have not been linked to the problem of matter-antimatter asymmetry.

## 6.1 Modified Dirac equation theories

There are several curved spacetime versions of the Dirac equation cf. [43–51] that have been proposed each with their unique taste and flavor in how it is arrived at. In our humble and modest view; save for the introduction of a seemingly mysterious four vector potential  $\mathcal{A}_\mu$ , what makes the curved spacetime version of the Dirac equations presented in Nyambuya [9] stand-out over other attempts is that the method used in arriving at these curved spacetime Dirac equations [9] is exactly the same as that used by Dirac [1, 2] in arriving at the Dirac equation. As will be demonstrated shortly in Section 7, this method used in Ref. [9] appears to us as the most straight forward and logical manner in which to arrive a curved spacetime version of the Dirac equation. All that has been done in Ref. [9], is to decompose the general Riemann metric  $g_{\mu\nu}$  in a manner that allows us to apply Dirac's [1, 2] prescription at arriving at the Dirac equation. Apart from this attempt [9]; attempts by e.g., Refs. cf. [43–51] that have been made to date, seek a curved spacetime version of the Dirac equation not from the fundamental curved spacetime energy-momentum equation  $(g_{\mu\nu}p^\mu p^\nu = m_0^2 c^4)$ , but take the Dirac equation as their point of departure. Our said approach (first presented in Ref. [9]) is new. In Section 7, we will present this new curved spacetime Dirac equation and proceed thereafter to demonstrate how this equation can be used to proffer a solution to this relatively long-standing matter-antimatter problem.

## 6.2 Modified Standard Model approach

According to Robson [52] who has developed a viable alternative to the SM-model called the Generation Model (GM) of particle physics [53–59], the matter-antimatter asymmetry problem can be solved within the framework of GM, where one can demonstrate that this asymmetry problem can be understood in terms of the composite leptons and quarks of the GM. According to Robson [52], one notes from this GM that there is essentially no matter-antimatter asymmetry in the present Universe and that the observed hydrogen-antihydrogen asymmetry may be understood in terms of statistical fluctuations associated with the complex many-body processes involved in the formation of either a hydrogen atom or an antihydrogen atom.

In Robson's [52] GM, the original antimatter created in the Big Bang is now contained within the stable composite Leptons, the Electrons and Neutrinos, and the stable composite quarks, the weak eigenstate up and down quarks that comprise the Protons and neutrons, within the hydrogen, helium and heavier atoms of the universe. Thus there is no matter-antimatter asymmetry in the present universe. However, there does exist a hydrogen-antihydrogen asymmetry where the present Universe consists predominantly of hydrogen atoms and virtually no antihydrogen atoms—and, as afore-stated—with the hydrogen-antihydrogen asymmetry understood in terms of statistical fluctuations associated with the complex many-body processes.

## 7. Curved spacetime Dirac equation

Our general feeling about the Dirac equation is that once it was discovered, it was taken up “very fast” and used as a most fundamental basis for building almost

all—if not all—aspects of QFTs. In the literature, there are no real visible efforts at an attempt on a curved spacetime version of the Dirac equation in the early stages of the Dirac's discovery of the Dirac equation. The major reason why the Dirac equation was taken up fast is that, at its birth—which was also the triumphant moment of its coronation and inauguration—it astoundingly explained, in a subtly natural and exquisitely brilliant manner, the quantum mystery of the origins of spin and as well, the then inexplicable gyromagnetic ratio of the Electron.

In the present section, we will give an exposition of the curved spacetime Dirac equation that we first presented in Ref. [9]. In this said attempt, the composite symmetric metric tensor of general relativity  $g_{\mu\nu}$  consisting of 10 potentials, is decomposed into a metric that can be described by a four vector potential  $\mathcal{A}_\mu$ , i.e.,  $g_{\mu\nu} = \mathcal{A}_\mu \mathcal{A}_\nu$ . Certainly, this is a reduction in complexity—i.e., from 10 potentials to only 4. At the time of our proposal, we were unsure how to interpret this description of the metric using a four vector because the only four vector potential that we know to describe a force is the Electromagnetic four vector potential.

However, if we are to still to the facts that  $g_{\mu\nu}$  describes the gravitational field, then, logic dictates that the gravitational field must be capable of being described by a four vector potential as suggested by this decomposition of the metric. Latter, it become clear that indeed, the gravitational field must be describable using a four vector potential and the proof of this has been given by Hera [60] and Behera [61] in their different theorems. We will not go into these theorems but hope that the reader will visit these very important proofs and convince themselves that indeed, the gravitational field must be describable (somehow) by a four vector potential. Assuming that gravity can be described by a four vector potential, we shall now proceed to present an exposition of the aforesaid curved spacetime Dirac equation that where first presented in Ref. [9]. What is new in this exposition is that, the gravitational four vector potential  $\mathcal{A}_\mu$  is consider for two cases, where in one case, these vector is a real function, while in the other, it is a complex function. It is in the case where this potential is complex that we are going to find the present proposed solution the matter-antimatter conundrum.

## 7.1 Real valued gravitational four vector potential

We shall begin by expanding  $g_{\mu\nu} p^\mu p^\nu$  into its 16 components—*albeit*, effectively 10 components and this is because of the symmetries of the metric, i.e.,  $g_{\mu\nu} = g_{\nu\mu}$ ; the four momentum space line element equation:  $g_{\mu\nu} p^\mu p^\nu = m_0^2 c_0^2$ , is such that:

$$\begin{aligned} g_{\mu\nu} p^\mu p^\nu = & g_{00} \left( \frac{E^2}{c_0^2} \right) + g_{11} p^1 p^1 + g_{22} p^2 p^2 + g_{33} p^3 p^3 \\ & + 2g_{01} p^0 p^1 + 2g_{02} p^0 p^2 + 2g_{03} p^0 p^3 \\ & + 2g_{12} p^1 p^2 + 2g_{13} p^1 p^3 + 2g_{23} p^2 p^3 = m_0^2 c_0^2. \end{aligned} \quad (16)$$

The terms sandwiched between  $g_{\mu\nu} p^\mu p^\nu$  and  $m_0^2 c_0^2$  are the expanded terms—the effective 10 components mentioned above. Upon canonical quantization, Eq. (16) becomes:

$$\begin{aligned} g_{\mu\nu} \partial^\mu \partial^\nu \Psi = & g_{00} \left( \frac{\partial_0^2}{c_0^2} \right) \Psi + g_{11} \partial^1 \partial^1 \Psi + g_{22} \partial^2 \partial^2 \Psi + g_{33} \partial^3 \partial^3 \Psi \\ & + 2g_{01} \partial^0 \partial^1 \Psi + 2g_{02} \partial^0 \partial^2 \Psi + 2g_{03} \partial^0 \partial^3 \Psi \\ & + 2g_{12} \partial^1 \partial^2 \Psi + 2g_{13} \partial^1 \partial^3 \Psi + 2g_{23} \partial^2 \partial^3 \Psi = \left( \frac{m_0 c_0}{\hbar} \right)^2 \Psi, \end{aligned} \quad (17)$$



where  $c_0$  is the speed of Light in *vacuo*. As first presented in Ref. [9]: in our quest for a curved spacetime Dirac equation which upon squaring would result in the above equation, we noted that if:  $g_{\mu\nu} \propto \mathcal{A}_\mu \mathcal{A}_\nu$  (which is mathematically and logically permissible), one can write down an equivalent curved spacetime Dirac equation. We noted that there are three configurations of the metric tensor,  $g_{\mu\nu}$ , that would do this. So, we decided to introduce a subscript for that would identify the metric tensor with the particular configuration, where in the new metric tensor, we now have  $g_{\mu\nu}^{(a)}$ , with  $a = (1, 2, 3)$  being the label of these three configurations. With the metric now written as  $g_{\mu\nu}^{(a)}$ , Eq. (17), now becomes:

$$\begin{aligned} g_{\mu\nu}^{(a)} \partial^\mu \partial^\nu \Psi = & g_{00}^{(a)} \left( \frac{\partial_0^2}{c_0^2} \right) \Psi + g_{11}^{(a)} \partial^1 \partial^1 \Psi + g_{22}^{(a)} \partial^2 \partial^2 \Psi + g_{33}^{(a)} \partial^3 \partial^3 \Psi \\ & + 2g_{01}^{(a)} \partial^0 \partial^1 \Psi + 2g_{02}^{(a)} \partial^0 \partial^2 \Psi + 2g_{03}^{(a)} \partial^0 \partial^3 \Psi \\ & + 2g_{12}^{(a)} \partial^1 \partial^2 \Psi + 2g_{13}^{(a)} \partial^1 \partial^3 \Psi + 2g_{23}^{(a)} \partial^2 \partial^3 \Psi = \left( \frac{m_0 c_0}{\hbar} \right)^2 \Psi, \end{aligned} \quad (18)$$

so that written without the expanded terms, the curved spacetime Klein-Gordon equation would be:

$$g_{\mu\nu}^{(a)} \partial^\mu \partial^\nu \Psi = \left( \frac{m_0 c_0}{\hbar} \right)^2 \Psi. \quad (19)$$

These three configurations [representing three configurations of spacetimes that we have called the (1) Quadratic Spacetime, (2) Parabolic Spacetime and (3) Hyperbolic Space, respectively] of the metric tensor are:

$$\left[ g_{\mu\nu}^{(1)} \right] = \begin{pmatrix} +\mathcal{A}_0 \mathcal{A}_0 & 0 & 0 & 0 \\ 0 & -\mathcal{A}_1 \mathcal{A}_1 & 0 & 0 \\ 0 & 0 & -\mathcal{A}_2 \mathcal{A}_2 & 0 \\ 0 & 0 & 0 & -\mathcal{A}_3 \mathcal{A}_3 \end{pmatrix} \mathcal{I}_4, \quad (20)$$

$$\left[ g_{\mu\nu}^{(2)} \right] = \begin{pmatrix} +\mathcal{A}_0 \mathcal{A}_0 & +\mathcal{A}_0 \mathcal{A}_1 & +\mathcal{A}_0 \mathcal{A}_2 & +\mathcal{A}_0 \mathcal{A}_3 \\ +\mathcal{A}_1 \mathcal{A}_0 & -\mathcal{A}_1 \mathcal{A}_1 & +\mathcal{A}_1 \mathcal{A}_2 & +\mathcal{A}_1 \mathcal{A}_3 \\ +\mathcal{A}_2 \mathcal{A}_0 & +\mathcal{A}_2 \mathcal{A}_1 & -\mathcal{A}_2 \mathcal{A}_2 & +\mathcal{A}_2 \mathcal{A}_3 \\ +\mathcal{A}_3 \mathcal{A}_0 & +\mathcal{A}_3 \mathcal{A}_1 & +\mathcal{A}_3 \mathcal{A}_2 & -\mathcal{A}_3 \mathcal{A}_3 \end{pmatrix} \mathcal{I}_4, \quad (21)$$

$$\left[ g_{\mu\nu}^{(3)} \right] = \begin{pmatrix} +\mathcal{A}_0 \mathcal{A}_0 & -\mathcal{A}_0 \mathcal{A}_1 & -\mathcal{A}_0 \mathcal{A}_2 & -\mathcal{A}_0 \mathcal{A}_3 \\ -\mathcal{A}_1 \mathcal{A}_0 & -\mathcal{A}_1 \mathcal{A}_1 & +\mathcal{A}_1 \mathcal{A}_2 & -\mathcal{A}_1 \mathcal{A}_3 \\ -\mathcal{A}_2 \mathcal{A}_0 & -\mathcal{A}_2 \mathcal{A}_1 & -\mathcal{A}_2 \mathcal{A}_2 & -\mathcal{A}_2 \mathcal{A}_3 \\ -\mathcal{A}_3 \mathcal{A}_0 & -\mathcal{A}_3 \mathcal{A}_1 & +\mathcal{A}_3 \mathcal{A}_2 & -\mathcal{A}_3 \mathcal{A}_3 \end{pmatrix} \mathcal{I}_4. \quad (22)$$

In a general, written in a condensed form, these three metric tensors  $g_{\mu\nu}^{(1)}$ ,  $g_{\mu\nu}^{(2)}$  and  $g_{\mu\nu}^{(3)}$ , are such that:

$$\left[ g_{\mu\nu}^{(a)} \right] = \begin{pmatrix} +\mathcal{A}_0 \mathcal{A}_0 & \lambda_a \mathcal{A}_0 \mathcal{A}_1 & \lambda_a \mathcal{A}_0 \mathcal{A}_2 & \lambda_a \mathcal{A}_0 \mathcal{A}_3 \\ \lambda_a \mathcal{A}_1 \mathcal{A}_0 & -\mathcal{A}_1 \mathcal{A}_1 & \lambda_a \mathcal{A}_1 \mathcal{A}_2 & \lambda_a \mathcal{A}_1 \mathcal{A}_3 \\ \lambda_a \mathcal{A}_2 \mathcal{A}_0 & \lambda_a \mathcal{A}_2 \mathcal{A}_1 & -\mathcal{A}_2 \mathcal{A}_2 & \lambda_a \mathcal{A}_2 \mathcal{A}_3 \\ \lambda_a \mathcal{A}_3 \mathcal{A}_0 & \lambda_a \mathcal{A}_3 \mathcal{A}_1 & \lambda_a \mathcal{A}_3 \mathcal{A}_2 & -\mathcal{A}_3 \mathcal{A}_3 \end{pmatrix} \mathcal{I}_4, \quad (23)$$

where—further: in a much more compact form, this metric can be written as:

$$g_{\mu\nu}^{(a)} = \frac{1}{2} \left( \mathcal{A}_\mu^{(a)} \mathcal{A}_\nu^{(a)} + \mathcal{A}_\nu^{(a)} \mathcal{A}_\mu^{(a)} \right) = \frac{1}{2} \left\{ \mathcal{A}_\mu^{(a)}, \mathcal{A}_\nu^{(a)} \right\} \in \mathbb{R}, \quad (24)$$

where:

$$\mathcal{A}_\mu^{(a)} = \gamma_\mu^{(a)} \mathcal{A}_\mu, \quad (25)$$

and the  $4 \times 4$  matrices  $\gamma_\mu^{(a)}$  are such that:

$$\gamma_0^{(a)} = \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix} = \gamma^0, \quad (26)$$

$$\gamma_k^{(a)} = \frac{1}{2} \begin{pmatrix} 2\lambda_a \mathcal{I}_2 & i^{\lambda_a |\lambda_a|} \sqrt{2^{|\lambda_a|}} \sigma_k \\ i^{\lambda_a |\lambda_a|} \sqrt{2^{|\lambda_a|}} \sigma_k & -2\lambda_a \mathcal{I}_2 \end{pmatrix}.$$

The  $\lambda_a$ 's in Eq. (26) are defined such that when:

$$a = \begin{cases} 1, & \text{then } (\lambda_1 = 0) : \text{ Quadratic Spacetime (QST).} \\ 2, & \text{then } (\lambda_2 = +1) : \text{ Parabolic Spacetime (PST).} \\ 3, & \text{then } (\lambda_3 = -1) : \text{ Hyperbolic Spacetime (HST).} \end{cases} \quad (27)$$

The index “ $a$ ” is not an active index as are the Greek indices. This index labels a particular curvature of spacetime, i.e., whether spacetime is flat,<sup>3</sup> positive or negatively curved as defined by the resulting metric  $g_{\mu\nu}^{(a)}$  which is given in Eq. (23). So, in the end, the resulting and desired curved spacetime Dirac equation is:

$$i\hbar \mathcal{A}_\mu^{(a)} \partial^\mu \psi = m_0 c_0 \psi, \quad (28)$$

where it is understood that  $\mathcal{A}_\mu$  is a real valued gravitational four vector function. In the subsequent section, we will show that the above equation with a complex valued gravitational four vector function violates  $C$ -symmetry. A violation of  $C$ -symmetry is all one needs to explain the whereabouts of the missing antimatter. Multiplication by  $(i\hbar \mathcal{A}_\nu^{(a)} \partial^\nu)$  from the left on the left handside of Eq. (28) and on the right by  $m_0 c_0$  and this is on the understanding that these operators are identical —i.e.,  $i\hbar \mathcal{A}_\nu^{(a)} \partial^\nu \equiv m_0 c_0 \mathcal{I}_4$ : one will lead to the curved spacetime Klein-Gordon equation provided:

$$\mathcal{A}_\nu^{(a)} \partial^\nu \mathcal{A}_\mu^{(a)} = 0. \quad (29)$$

Therefore, the above equation enters into this theory as a gauge condition to be met by the real-valued gravitational four-vector potential  $\mathcal{A}_\mu^{(a)}$ .

<sup>3</sup> By *flat*, it here is not meant that the spacetime is Minkowski *flat*, but that the metric has no off diagonal terms. On the same footing, by *positively curved spacetime*, it meant that metric has positive off diagonal terms and likewise, a *negatively curved spacetime*, it meant that metric has negative off diagonal terms.

## 7.2 Complex valued gravitational four vector potential

We shall now assume that the gravitational four vector field is complex valued and we shall denote it as:  $\tilde{\mathcal{A}}_\nu^{(a)}$  and  $\tilde{\mathcal{A}}_\nu$ . In this event where we have a complex valued gravitational four vector field, the metric will have to be defined as:

$$g_{\mu\nu}^{(a)} = \frac{1}{2} \left( \tilde{\mathcal{A}}_\mu^{(a)\dagger} \tilde{\mathcal{A}}_\nu^{(a)} + \tilde{\mathcal{A}}_\nu^{(a)\dagger} \tilde{\mathcal{A}}_\mu^{(a)} \right) = \frac{1}{2} \left\{ \tilde{\mathcal{A}}_\mu^{(a)\dagger}, \tilde{\mathcal{A}}_\nu^{(a)} \right\}, \quad (30)$$

where—as before:

$$\tilde{\mathcal{A}}_\mu^{(a)} = \tilde{\gamma}_\mu^{(a)} \mathcal{A}_\mu, \quad (31)$$

and the new  $4 \times 4$   $\tilde{\gamma}$ -matrices are such that:

$$\tilde{\gamma}_0^{(a)} = \begin{pmatrix} 0 & \mathcal{I}_2 \\ -\mathcal{I}_2 & 0 \end{pmatrix}, \quad \tilde{\gamma}_k^{(a)} = \frac{1}{2} \begin{pmatrix} i^{\lambda_a |\lambda_a|} \sqrt{2|\lambda_a|} \sigma_k & 2\lambda_a \mathcal{I}_2 \\ -2\lambda_a \mathcal{I}_2 & i^{\lambda_a |\lambda_a|} \sqrt{2|\lambda_a|} \sigma_k \end{pmatrix}. \quad (32)$$

Written in full as is the case in Eq. (23), the metric  $g_{\mu\nu}^{(a)}$  for a complex valued gravitational four vector potential  $\mathcal{A}_\mu$  is such that:

$$[g_{\mu\nu}^{(a)}] = \begin{pmatrix} +\mathcal{A}_0^\dagger \mathcal{A}_0 & \lambda_a \mathcal{A}_0^\dagger \mathcal{A}_1 & \lambda_a \mathcal{A}_0^\dagger \mathcal{A}_2 & \lambda_a \mathcal{A}_0^\dagger \mathcal{A}_3 \\ \lambda_a \mathcal{A}_1^\dagger \mathcal{A}_0 & -\mathcal{A}_1^\dagger \mathcal{A}_1 & \lambda_a \mathcal{A}_1^\dagger \mathcal{A}_2 & \lambda_a \mathcal{A}_1^\dagger \mathcal{A}_3 \\ \lambda_a \mathcal{A}_2^\dagger \mathcal{A}_0 & \lambda_a \mathcal{A}_2^\dagger \mathcal{A}_1 & -\mathcal{A}_2^\dagger \mathcal{A}_2 & \lambda_a \mathcal{A}_2^\dagger \mathcal{A}_3 \\ \lambda_a \mathcal{A}_3^\dagger \mathcal{A}_0 & \lambda_a \mathcal{A}_3^\dagger \mathcal{A}_1 & \lambda_a \mathcal{A}_3^\dagger \mathcal{A}_2 & -\mathcal{A}_3^\dagger \mathcal{A}_3 \end{pmatrix} \mathcal{I}_4. \quad (33)$$

With the  $\tilde{\gamma}$ -matrices defined; in-order for  $g_{\mu\nu}^{(a)} \in \mathbb{R}$ , the gravitational four vector potential will have to be defined in the *de Broglie-Bohm* [62–64] polar form as follows:

$$\mathcal{A}_\mu = \phi_\mu \exp \left( \frac{i\mathcal{S}}{\hbar} \right), \quad (34)$$

where:  $\phi_\mu = \phi_\mu(\mathbf{r}, t) \in \mathbb{R}$ , is a differentiable, uniform continuous, and integrable four-vector-valued function;  $\mathcal{S} = \mathcal{S}(\mathbf{r}, t) \in \mathbb{R}$ , is a zero rank scalar that is also a differential, uniform continuous and integrable function. With  $\mathcal{A}_\mu$  defined as it defined above, it follows that:

$$\tilde{\mathcal{A}}_\mu^* \tilde{\mathcal{A}}_\nu = \tilde{\mathcal{A}}_\nu^* \tilde{\mathcal{A}}_\mu = \phi_\mu \phi_\nu \in \mathbb{R}, \quad (35)$$

hence:

$$g_{\mu\nu}^{(a)} = \frac{1}{2} \left\{ \tilde{\mathcal{A}}_\mu^{(a)\dagger}, \tilde{\mathcal{A}}_\nu^{(a)} \right\} = \frac{1}{2} \left\{ \tilde{\gamma}_\mu^{(a)\dagger}, \tilde{\gamma}_\nu^{(a)} \right\} \tilde{\mathcal{A}}_\mu^* \tilde{\mathcal{A}}_\nu = \frac{1}{2} \left\{ \tilde{\gamma}_\mu^{(a)\dagger}, \tilde{\gamma}_\nu^{(a)} \right\} \phi_\mu \phi_\nu \in \mathbb{R}. \quad (36)$$

So, in the end, the resulting and desired curved spacetime Dirac equation is:

$$i\hbar \tilde{\mathcal{A}}_\mu^{(a)} \partial^\mu \psi = m_0 c_0 \psi, \quad (37)$$

where it is understood that  $\tilde{\mathcal{A}}_\mu$  is to be a complex valued gravitational four vector function. In the subsequent section, we will show that the above equation

with a complex valued gravitational four vector function violates  $C$ -symmetry. A violation of  $C$ -symmetry is all one needs to explain the whereabouts of the missing antimatter. Multiplication by  $\left(i\hbar\mathcal{A}_\nu^{(a)}\partial^\nu\right)^\dagger$  from the left on the left handside of Eq. (37) and on the right by  $(m_0c_0)^\dagger$  and this is on the understanding that these operators are identical—i.e.,  $\left(i\hbar\mathcal{A}_\nu^{(a)}\partial^\nu\right)^\dagger \equiv (m_0c_0)^\dagger\mathcal{I}_4$ : one will be led to the curved spacetime Klein-Gordon equation provided:

$$\tilde{\mathcal{A}}_\nu^{(a)\dagger}\partial^\nu\tilde{\mathcal{A}}_\mu^{(a)}=0. \quad (38)$$

Therefore—as before, i.e., as in Eq. (29), the above Eq. (38) enters into this theory as a *gauge condition* to be met by the gravitational four vector potential  $\tilde{\mathcal{A}}_\mu^{(a)}$ .

## 8. Symmetries of the curved spacetime Dirac equation

We will demonstrate that the complex valued gravitational four vector potential curved spacetime Dirac Eq. (37) violates  $C$ -symmetry. To that end, we shall start off in the usual manner by placing the curved spacetime Dirac particle  $\psi$  inside an external magnetic field whose electromagnetic four vector potential is  $A_{ex}^\mu$ . So doing, Eq. (37) will transform to:

$$i\hbar\tilde{\mathcal{A}}_\mu^{(a)}(\partial^\mu+ieA_{ex}^\mu)\psi=m_0c_0\psi. \quad (39)$$

Now, we will have to switch the external magnetic field by reversing the electromagnetic four vector potential is  $A_{ex}^\mu$ , i.e.,  $A_{ex}^\mu \mapsto -A_{ex}^\mu$ . So doing, Eq. (39) will transform to:

$$i\hbar\tilde{\mathcal{A}}_\mu^{(a)}(\partial^\mu-ieA_{ex}^\mu)\psi=m_0c_0\psi. \quad (40)$$

Now, if Eq. (37) is symmetric under electrical charge conjugation, there must exist a set of permissible mathematical operations that when applied to Eq. (40), they will lead us back to Eq. (39). The first such permissible mathematical operations is to apply the complex-operation on both-sides of Eq. (39): this complex operation will restore the sign in the coefficient of  $A_{ex}^\mu$ , i.e.,

$$-i\hbar\tilde{\mathcal{A}}_\mu^*\tilde{\gamma}_\mu^{(a)*}(\partial^\mu+ieA_{ex}^\mu)\psi^*=m_0c_0\psi^* \quad (41)$$

Now, in-order to revert back to Eq. (39), we need to find a set of permissible mathematical operations that will remove the complex operation on the terms:  $\tilde{\mathcal{A}}_\mu^*$  and  $\tilde{\gamma}_\mu^{(a)*}$ . We can remove the complex-operation on  $\tilde{\gamma}_\mu^{(a)*}$  because of the following *Algebra*:

$$\gamma_0\gamma_2\tilde{\gamma}_\mu^{(a)*}=-\tilde{\gamma}_\mu^{(a)}\gamma_0\gamma_2. \quad (42)$$

The removal of the complex-operation on  $\tilde{\gamma}_\mu^{(a)*}$  is achieved by multiplying on both-sides of Eq. (41) by  $\gamma_0\gamma_2$ , i.e.,

$$-i\hbar\tilde{\mathcal{A}}_\mu^*\gamma_0\gamma_2\tilde{\gamma}_\mu^{(a)*}(\partial^\mu+ieA_{ex}^\mu)\psi^*=m_0c_0\gamma_0\gamma_2\psi^*, \quad (43)$$



and using the fact Eq. (42), it follows that Eq. (43) will reduce to:

$$i\hbar\tilde{\mathcal{A}}_\mu^*\tilde{\gamma}_\mu^{(a)}(\partial^\mu + ieA_{ex}^\mu)(\gamma_0\gamma_2\psi^*) = m_0c_0(\gamma_0\gamma_2\psi^*). \quad (44)$$

Now, Eq. (44) can be re-written as:

$$i\hbar\tilde{\mathcal{A}}_\mu^*\tilde{\gamma}_\mu^{(a)}(\partial^\mu + ieA_{ex}^\mu)\psi_c = m_0c_0\psi_c, \quad (45)$$

where:  $\psi_c = \gamma_0\gamma_2\psi^*$ , is the antiparticle. Now, in-order for the above equation to revert back to the original Eq. (39), there is need for the gravitational four vector,  $\tilde{\mathcal{A}}_\mu$ , to be real, i.e.,  $\tilde{\mathcal{A}}_\mu^* = \tilde{\mathcal{A}}_\mu$ . If this condition ( $\tilde{\mathcal{A}}_\mu^* = \tilde{\mathcal{A}}_\mu$ ) cannot be met because  $A_\mu$  is a complex valued function, then, the curved spacetime Dirac equation is not symmetric under charge conjugation, hence it will violate  $C$ -symmetry.

## 9. General discussion

Tremendous effort and thrust has been put on experimental and observational attempts whose aim is to procure the necessary evidence to support Sakholov's [8] hypothesis of the *sine-quo-non* conditions needed to be met in-order to explain the clearly obvious matter dominance observed in the Universe. Little or no effort has been put—let alone suggested, that, perhaps, the fault (solution) may lay in the very *Physical Law* that we have used to probe and understand the Universe and this law is the all-symmetric Dirac equation. It is quite understandable why this may be the case—the Dirac equation is so successful so much that, it is easy to be “blinded” by this success to an extent that one cannot—with *suspicion*—point the “little prickling finger” at it. In the present chapter, we have had to gather the necessary courage and temerity to do just that.

From what has been presented above, it is clear that from a theoretical standpoint, all one would need in-order to explain the missing antimatter is to proceed and henceforth make the hypothesis that the gravitational four vector,  $\mathcal{A}_\mu$ , is a complex field. This would mean that during the moment of creation, either matter is produced, with no antimatter, or, antimatter is produced, with no matter! Thus, in the framework of the foregoing curved spacetime version of the Dirac equation, the Universe is pristinely asymmetric in its matter-antimatter constitution right from the moment of creation. There would be no need to have the Sakholov conditions, or, any other exogenous mechanism or condition in-order for one to explain the matter-antimatter asymmetry. This alternative way at looking at this long standing problem appears to be the simplest way out of this ponderous and vexing conundrum of the missing antimatter. To accept this solution requires one to accept the proposed curved spacetime Dirac equations.

Sakholov's [8] hypothesis starts off by accepting the Dirac Eq. (3) in its bare form, the meaning of which is that it assumes a perfectly symmetric Universe which then proceeds to become asymmetric once the Sakholov's [8] conditions are met. Sakholov's [8] conditions require  $C$ ,  $CP$ ,  $B$ -number violating processes and the existence of nonthermodynamic equilibrium. The point here is that—what is needed is that a section of the symmetric matter-antimatter soup meets Sakholov's [8] criterion of having these processes and once this is the case, the Universe can then proceed from a state with:  $B = 0$ , to a state with:  $B \neq 0$ . In a perfectly matter-antimatter symmetric Universe, these processes may require certain physical conditions of energy and temperature in-order to trigger them, thus leading to a matter-antimatter asymmetric Universe.

However, in the suggestion being made here-in, we envisage a Universe where matter is created *via* the  $C$ -symmetry violating curved spacetime Dirac Eq. (37) where the Universe is created containing only matter and no antimatter and this will come about because of the phase factor in the gravitational four vector field:  $\mathcal{A}_\mu^{(a)} = \phi_\mu e^{iS/\hbar}$ ; that is to say, for so long as at the moment of creation, this phase factor is not equal to zero  $S \neq 0$ , the Universe will be completely asymmetric in its matter-antimatter constitution. Even if the Universe were to evolve from a state with:  $S \neq 0$ , to a state with:  $S = 0$ , at a latter time in its evolution, the Universe will—*throughout its entire evolution*—still be asymmetric in its matter-antimatter constitution. In the end, no experiments will be needed to find these  $C$ ,  $CP$ ,  $B$ -number violating processes. All we would need is to test the curved spacetime Dirac equation where our matter-antimatter asymmetry is being championed.

In-closing: insofar as accepting the proposed curved spacetime Dirac Eq. (37), it is important to note that the way these equations have been “derived” is exactly the same-way Dirac arrived at his equation. All we have done in this proposed curved spacetime Dirac Eq. (37) is to note that the metric tensor of spacetime  $g_{\mu\nu}$ , can be decomposed in such a manner that at its most fundamental and simplest level, it can be represented by a four vector  $\mathcal{A}_\mu$ . This gravitational four vector potential,  $\mathcal{A}_\mu$ , will have to represent the gravitational field. On this, one may object because the GTR—which is not only the current best model of gravitation, but the most successful model of gravitation; describes gravity as tensor field that is represented by not four, but 10 potentials.

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